

Testing QED and atomic-nuclear interactions with high- Z ions

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Abstract

Measurements of high- Z hydrogen-like ions have not yet been able to compete with the ultra-high precise test of quantum electrodynamics (QED) carried out using atomic hydrogen. However, recent advances in theory and experiment have enabled measurements involving high- Z lithium-like ions to test two-loop QED at a level similar to that achieved with atomic hydrogen. Tests of two-loop QED are limited in both cases by the uncertainties in the finite nuclear size, i.e. the uncertainties associated with the proton charge radius and the finite size of high- Z nuclei, respectively. Future experiments employing high- Z ions are described that might go beyond the present limitations. Measurements of the hyperfine splitting in highly charged ions cannot readily be described by theory, in part because of poor understanding of the finite nuclear magnetization radius. The discrepancy between measurements and theory leaves open the possibility of new physics not yet addressed by our current understanding of atomic–nuclear interactions.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The theory of quantum electrodynamics has been extremely successful in guiding our understanding of nature. It has been validated in numerous experiments, and many of these measurements have achieved enormous precision. For example, the anomalous magnetic moment of the electron ($g - 2$) has been measured in Penning traps at the few parts per billion level 20 years ago [1] and more recently even at the few parts per trillion level [2]. Quantum electrodynamic (QED) calculations of up to four-loop Feynman diagrams (equivalent to fourth order in the fine-structure constant α) have been carried out to match the precision of the experiments [3]. The precision of the measurements and calculations exceeds that with which even the fundamental constants are known. As a result, the measurements of the $g - 2$ value of the free electron (or positron) can in principle be inverted and used to determine the value of the fine-structure constant $\alpha = 2\pi e^2/hc$ with an accuracy that is higher than is afforded by any other measurement [4] provided; of course, to do so we have to assume that the predictions of QED theory are correct.

The QED description of a bound atomic electron, i.e. of a system in which the electron no longer moves in a vacuum

but in the (strong) electric field of a nucleus, is considerably more complex than that describing a free electron. Decisive experimental verifications of the predictions of QED in a bound system have mainly concentrated on neutral atoms and a few low- Z ions, i.e. atomic systems accessible with laser spectroscopy. The reason is that laser spectroscopy represents an ultra-precise experimental tool. Indeed, laser spectroscopy of transitions in atomic hydrogen that are sensitive to QED effects has achieved tremendous precision. The energy of the 1s–2s transition in hydrogen, for example, has been measured with 14-digit accuracy [5]. Indeed, the 1 part in 10^{14} frequency measurement of the 1s–2s transition in atomic hydrogen represents the most accurate measurement of any optical frequency. The measurement is used to determine a highly precise value of the Rydberg constant [5, 6]. In fact, it is so precise that it may be used to test the variation of the fundamental constants over time [7–9]. This measurement also allows a test of the predictions of QED with utmost precision, surpassing that of a direct, radio-frequency measurement of the 2s–2p Lamb shift [10].

One-loop QED effects (vacuum polarization and self energy) enter for atomic hydrogen at a level that requires a precision of ‘only’ six digits. The measured QED contribution

to the 1s level in hydrogen is 8172.876(29) MHz out of a total 1s–2s transition energy of 2466 061.413 GHz [6]. A global average of the measurements from both the Garching group and the Paris group [6, 11, 12] gives an even better value of 8172.840(22) MHz. This means that the 1s–2s measurement of atomic hydrogen determines QED effects at the level of 2.7 parts in 10^6 .

Faced with the high precision achieved in atomic hydrogen, tests of QED using high-Z ions have taken a back seat. The best measurement of the 1s Lamb shift of hydrogen-like uranium, U^{91+} , achieved so far an accuracy of 4.5 eV [13]. This compares to the calculated QED contributions of about 266 eV. Thus, these measurements test QED theory at a level of 1.7 parts in 10^2 , which is orders of magnitude less than that in the test of QED effects at the level of 2.7 parts in 10^6 using atomic hydrogen discussed above. A recent review [14] of 42 measurements of the QED contributions to the 1s level of high-Z hydrogen-like ions showed that not a single measurement disagreed with QED theory outside the $1-\sigma$ experimental error bar. From purely statistical arguments, about one-third of the measurements, or about 14 of 42 measurements, should have produced an answer that differed from predictions. We take this to mean that experimenters do not want to disagree with a theory that has been so successful and tested on a much deeper level with atomic hydrogen than possible in these 42 measurements involving high-Z hydrogen-like ions. But the corollary to this statement, which is debatable, is that as a whole these measurements did not contribute to a better understanding of QED theory.

QED effects in ions with more than one electron are much more difficult to calculate because the additional electrons ‘screen’ the nuclear potential. Early calculations of QED in multi-electron high-Z ions were estimates based on scaling the values calculated from first principles for hydrogen-like systems, as discussed recently by Cheng *et al* [15]. Thus, early ‘tests’ of QED with multi-electron systems were more of a test of the accuracy of the approximations used in the estimation of the QED energies than of *ab initio* calculations of the actual QED terms. This situation has dramatically improved since, as new theoretical methods, notably relativistic many-body perturbation theory (RMBPT) and relativistic configuration interaction (RCI) calculations, have been developed to calculate both Dirac and QED energies, including the correlation terms [15]. Excellent agreement between theory and experiment is now obtained, not only for alkali-like ions (which are similar to hydrogen-like ions in that they have a single valence electron outside a closed electron shell) but also for ions with multiple electrons in a given valence shell, such as beryllium-like, boron-like, carbon-like, magnesium-like, aluminum-like, silicon-like and zinc-like ions [15, 16]. These immense successes now allow testing of QED at a level that in some cases is roughly up to an order of magnitude more precise than has been accomplished in the best measurements of hydrogen-like uranium mentioned above. For example, RCI calculations allow a theoretical determination of the QED terms contributing to the $3s_{1/2}$ – $3p_{3/2}$ transition energy in sodium-like uranium, U^{81+} , with an estimated accuracy of 0.07 eV [17]. Thus, the 1305.12(2) eV

measurement of this transition on the Livermore SuperEBIT electron beam ion trap [18] provides a test of the 10.2 eV QED contribution at the level of 6.8 parts in 10^3 . This is about three times better than the best test of QED carried out in one-electron U^{91+} . In the one-electron measurements the accuracy with which QED can be tested is limited by the accuracy of the measurement, while the theoretical predictions of QED are all *ab initio*; in the multi-electron measurements the accuracy with which QED can be tested is limited by the accuracy of the theoretical predictions, as they struggle with accurate accounting of screening and correlation caused by multiple, interacting electrons, including the contributions from correlations with negative energy states [15]. The higher level of accuracy achieved in testing QED in the comparison of calculations and measurements of high-Z multi-electron system is, however, still orders of magnitude smaller than the test provided by atomic hydrogen.

Our discussion so far makes it seem that tests of QED with high-Z ions cannot compete with those performed on atomic hydrogen, and until a few years ago this has been true. However, there are three reasons why this is *no longer* true. First, a rigorous framework for calculating the Feynman diagrams arising from a fundamental QED approach, called *S*-matrix theory, has been developed, which allows one to make unambiguous theoretical predictions for testing with measurements [19]. This framework has been successfully applied to high-Z three-electron, i.e. lithium-like, ions. Second, highly accurate measurements of the $2s_{1/2}$ – $2p_{1/2}$ and the $2s_{1/2}$ – $2p_{3/2}$ transitions have become available. Third, these measurements no longer aim to test one-loop QED but are now sensitive to *two-loop* QED. In other words, measurements and calculations of high-Z ions nowadays go beyond trying to test one-loop QED, which is very well established by the ultraprecise laser measurements of neutral hydrogen. They now have entered a regime in which they begin testing the two-loop diagrams in the Feynman representation of QED (see figure 1). In this regime, measurements of high-Z ions can effectively compete with ultraprecise measurements of neutral hydrogen, and they are helped by the fact that as a fraction of the total, the two-loop contribution increases rapidly with the atomic number. In fact, tests of bound-state QED nowadays need to focus on two-loop QED. The reason is that if any breakdown of QED theory were to occur, it would likely be observable in higher order, based on what we already know of first-order QED. In the following, we will discuss these three points in detail.

We should note that recent comparisons of theory and measurement in atomic helium were grossly divergent, casting doubt on theoretical predictions. In this case, two independent calculations [20, 21] agreed with each other, but differed by as much as 24(!) standard deviations from several independent measurements [22–25], which in turn all agree well among themselves. High-Z measurements and theory can clearly add independent insight into such tantalizing issues, as they arise.

We will also give a brief discussion of the hyperfine structure measurements in high-Z ions. Although the hyperfine structure has been measured with extreme precision in neutral hydrogen [26], such measurements in high-Z ions represent a

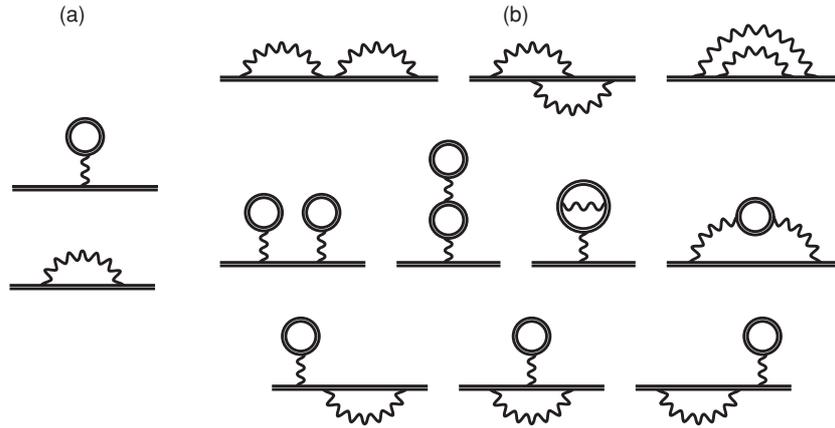


Figure 1. Feynman representation of QED: (a) one-loop self-energy and vacuum-polarization diagrams; (b) two-loop diagrams.

valuable probe of the magnetic and electric fields inside the nucleus. By measuring the hyperfine structure in different ions from the same isotope, atomic–nuclear interactions will be rigorously quantified, which means that such measurements will ultimately represent a new probe of QED in very strong magnetic fields. At present, such measurements have produced more questions than answers, making this a very exciting field of investigation.

2. Testing two-loop diagrams in neutral hydrogen

As mentioned in the introduction, the QED contribution to the 1s level in hydrogen is about 8173 MHz [6]. It turns out that essentially all of this energy is associated with the two one-loop diagrams shown in figure 1(a). QED calculations for neutral hydrogen have been successful in evaluating nested loops, such as those illustrated in figure 1(b) [27]. The two-loop contributions are predicted to enter at the 700 kHz level [28]. Thus, two-loop QED represents about 9 parts in 10^5 of the total 1s QED energy of atomic hydrogen. This compares to the 22 kHz precision of the laser measurement, which thus allows a measurement of the two-loop QED terms at the 3% level.

At this level of precision the finite size of the proton greatly modifies the theoretical predictions. The extended size of the proton dilutes the field that otherwise would be associated with a point charge. The uncertainty in a single proton radius measurement, e.g. $r = 0.862(12)$ fm [29], introduces an uncertainty of 32 kHz in the theoretical predictions [30]. This uncertainty limits a test of two-loop QED to an accuracy of 4.6%. However, several measurements of the proton radius have been made [29, 31–34], and these do not agree within their respective error bars. The spread in the different proton measurements introduces an uncertainty of 152 kHz. This uncertainty is about 22% of the predicted size of the two-loop QED contribution. The uncertainty would be almost twice as large if additional proton radius data were included in the spread that now have been discounted. Thus, until more decisive measurements of the proton radius are made [21], two-loop QED can be tested in atomic hydrogen only at the 22% level. A re-analysis of the world data on the proton

radius was carried out by Sick [35], which produced a value of $r = 0.890(18)$ fm. If this value were adopted, two-loop QED would have been tested at the 48 kHz level, or roughly at the 7% level. An overview of the relative precision with which two-loop QED is tested with measurements of atomic hydrogen is shown in figure 2.

3. Testing two-loop diagrams in hydrogen-like uranium

The two-loop QED contribution in atomic hydrogen is approximately 9 parts in 10^5 of the total QED energy. If that scaling were carried over to uranium, the two-loop part would be about 0.3 meV out of a total of 266 eV. Given the current experimental precision of 4.6 eV, a more than four orders of magnitude improvement in the experimental precision would be needed before an experiment would be sensitive to the two-loop QED terms.

But as we mentioned in the introduction, the fractional contribution of the two-loop QED energy to the total increases very rapidly with atomic number. QED calculations have recently been successful in evaluating the two-loop contributions for hydrogen-like uranium [45, 46]. The resulting theoretical value for the two-loop QED contributions to the 1s level in U^{91+} is 1.26(33) eV [46]. This is 0.5% of the total 1s QED energy of hydrogen-like uranium.

It is this very strong scaling of the two-loop QED contribution that makes experiments involving high- Z ions competitive with the ultraprecise measurements of atomic hydrogen. Two-loop contributions are much more prominent in high- Z ions than in atomic hydrogen, placing a less stringent requirement on the experimental precision to test the associated physics. In other words, while measurements of high- Z ions cannot compete with the tests of one-loop QED employing atomic hydrogen, they can compete in testing the physics of two-loop QED in bound systems.

The 4.6 eV accuracy achieved so far in the direct measurement of the U^{91+} 1s QED contribution has obviously not been sufficient to test the two-loop calculations. This can be seen in figure 2, where the experimental precision is more than 100% of the value of the two-loop QED. However, the

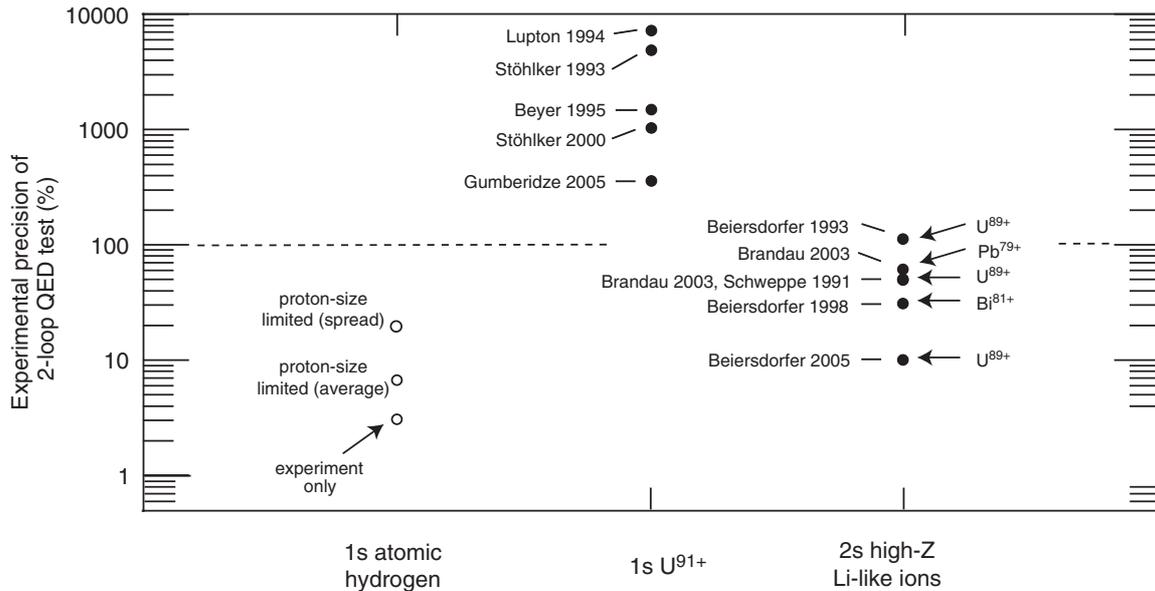


Figure 2. Experimental precision as a fraction (in per cent) of the respective two-loop QED contributions measured in atomic hydrogen, hydrogen-like U^{91+} and in the three highest-Z lithium-like ions measured to date. For atomic hydrogen, the precision limited by the uncertainty of the proton radius measurements is also shown as given by the uncertainty of the world-average value and by the spread in the three most reliable measurements. The U^{91+} results are from [13, 36–39]; the lithium-like ion results are from [40–44]. The experimental precision to test two-loop QED has to be smaller than 100% of the size of the two-loop term, which is indicated by the horizontal dashed line.

Table 1. Theoretical contributions [19] to the 1s Lambshift in hydrogen-like uranium and comparison with the two-loop QED value inferred from the measurement of U^{89+} [44] (in eV).

Contributing term	Value
Theory	
Finite nuclear size	198.81
One-loop QED	266.45
Recoil	0.51
Two-loop QED	-1.26(33)
Experiment	
U^{89+} measurement	-1.27(45)

recent measurement of the $2s_{1/2}-2p_{1/2}$ transition in lithium-like U^{89+} on the Livermore SuperEBIT electron beam ion trap [44], which we discuss in more detail below, allowed us to make the first such test of two-loop QED in hydrogen-like U^{91+} . As shown in [44, 47] and summarized in table 1, the measurement gives a value of 1.27(45) eV for the two-loop contribution to the 1s level. This is a test of the predicted two-loop contribution at the 33% level. This is already competitive with the two-loop test in atomic hydrogen, which is currently possible at the 7–22% level, depending on the assumptions concerning the proton radius.

4. Testing two-loop diagrams in lithium-like uranium

The Lamb shift in atomic hydrogen is defined as the energy splitting of the $2s_{1/2}$ and $2p_{1/2}$ levels. Without QED and finite nuclear size effects these two levels would have the same energy. A measurement of the splitting of these levels in a high-Z hydrogen-like ion, thus, is a direct measurement of the

Table 2. Measurements of the $2s_{1/2}-2p_{1/2}$ transition energy (in eV).

Year	Value	Reference
1991	280.59(10)	[40]
2003	280.516(99)	[43]
2005	280.645(15)	[44]

QED and finite nuclear size effects. Probing this interval in high-Z hydrogen-like ions, however, is difficult for a variety of reasons, not the least of which is the fact that the $2p_{1/2}$ level in hydrogen-like U^{91+} will rapidly (with a radiative rate of $A_r \approx 5 \times 10^{16}$ 1/s) decay to the $1s_{1/2}$ ground state, and such a measurement has not yet been accomplished. By contrast, a measurement of the splitting of the $2s_{1/2}$ and $2p_{1/2}$ levels in lithium-like ions has been possible. Here the electron in the $2s_{1/2}$ state cannot decay further, and the width of the $2p_{1/2}$ level, which decays at a rate of about $A_r \approx 2 \times 10^{10}$ 1/s, is merely about 0.01 meV. However, the splitting in lithium-like ions is not solely due to the Lamb shift. The Dirac interaction with the two electrons in the $1s^2$ shell and nuclear size effects account for the bulk of the splitting, or about 85% of the total 280.6 eV splitting in U^{89+} .

Experimentally, the energy splitting of the $2s_{1/2}$ and $2p_{1/2}$ levels in high-Z lithium-like ions has been measured in several instances, as summarized in table 2 for U^{89+} and shown in figure 3. The first measurement of U^{89+} was performed in 1991 and determined a value of 280.59 eV [40]. The accuracy achieved was 0.1 eV. A subsequent measurement more than a decade later found a somewhat lower value, 280.530(99) eV, but did not improve on the error bar [43]. A breakthrough was achieved in 2005, when the $2s_{1/2}-2p_{1/2}$ transition in U^{89+} was measured with 0.015 eV accuracy [44]. In principle, all

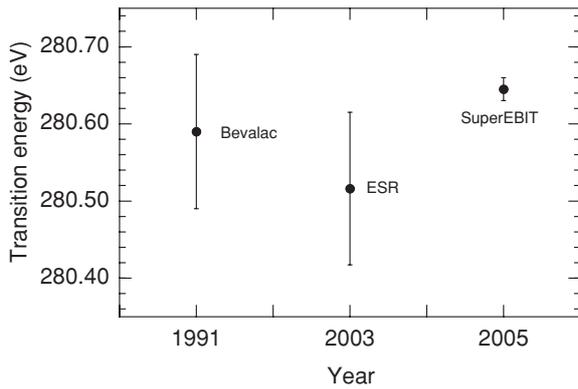


Figure 3. Measurements of the $2s_{1/2}-2p_{1/2}$ transition in U^{89+} [40, 43, 44].

three measurements are sensitive to the two-loop QED term, as illustrated in figure 1. However, only the newest experimental value has been able to provide a test of the two-loop QED terms, as the following discussion will explain.

By the time the 2005 measurement was made, *ab initio*, Feynman-diagram-based methods to evaluate energy levels in high- Z lithium-like ions had sufficiently advanced to calculate first-order terms, including screening and photon exchange terms (see the review by Sapirstein and Cheng [19]). The latter are very important for a multi-electron system such as lithium-like uranium. Those terms do not exist in atomic hydrogen or hydrogen-like ions. Once those terms could be calculated reliably from first principles, testing two-loop QED in lithium-like systems became feasible. The main uncertainty in 2005 derived from the value of the three-photon exchange term, which was only estimated from the previous-order term and thus carried an uncertainty of 0.07 eV [48]. As a result, the total calculated energy for the $2s_{1/2}-2p_{1/2}$ transition in U^{89+} , but excluding two-loop QED, was calculated by Andreev *et al* [48], for example, to be 280.47(7) eV. Sapirstein and Cheng and Yerokhin *et al* determined 280.43(7) and 280.44(10) eV, respectively (see [44]).

The combination of the values and uncertainties of the experimental $2s_{1/2}-2p_{1/2}$ transition energy in U^{89+} compared to the size and uncertainty of the calculated values left little room for determining the two-loop QED contribution from the measurements made prior to 2005. The 2005 measurement of the $2s_{1/2}-2p_{1/2}$ transition energy in U^{89+} , however, reduced the experimental error to the point where it was negligible compared to the theoretical uncertainties, and the two-loop contribution to the $2s$ level could be inferred. The resulting value was $-0.23(7)$ eV (and $-0.20(7)$ eV for the two-loop contribution to the $2s_{1/2}-2p_{1/2}$ transition energy) [44]. This value is in complete agreement with a subsequent calculation of the two-loop terms by Yerokhin *et al* [49], as summarized in table 3.

The 0.015 eV uncertainty of the recent U^{89+} measurement in principle can test two-loop QED to 6.5%. This is better than the proton-size-limited test performed with atomic hydrogen. However, the fact that there are currently 0.07 eV uncertainties in the higher order photon exchange terms limits the test to

Table 3. Theoretical contributions [54] to the two-loop QED contribution to the $2s_{1/2}-2p_{1/2}$ transition energy in lithium-like uranium and comparison with the value inferred from the measurement of U^{89+} [44] (in eV).

Contributing term	Value
Theory	
SESE ^a	0.30
VPVP ^b	0.14(5)
S(VP)E ^c	-0.02(5)
SEVP ^d	-0.19
Sum	0.23(7)
Experiment	
U^{89+} measurement	0.20(7)

^a Top three diagrams in figure 1(b).

^b Last diagram in the middle of figure 1(b).

^c Bottom three diagrams in figure 1(b).

^d First three diagrams in the middle of figure 1(b).

about 30%. In principle, this limitation can, and most likely will, be reduced in the future, when *ab initio* calculations become available. The finite nuclear size of uranium, however, is also an issue that limits the accuracy with which two-loop QED can be tested in uranium. The value of the nuclear radius $\langle r^2 \rangle = 5.860(2)$ fm given by Zumbro *et al* [50] limits the accuracy with which the Dirac energy of the $2s_{1/2}-2p_{1/2}$ transition in U^{89+} can be calculated to about 0.02 eV. Various size measurements have been evaluated by Angeli [51], and a value of $\langle r^2 \rangle = 5.851(7)$ fm was given. This limits the accuracy with which the Dirac energy can be calculated to about 0.09 eV. The accuracy with which two-loop QED can be tested due to the uncertainty in the finite size of the uranium nucleus is thus limited to about 40%. This limitation is disappointingly high. On the other hand, it is comparable to the limitations experienced with atomic hydrogen.

The limitation imposed by the uncertainty in the size of the uranium nucleus carries over to tests of the $1s$ QED in hydrogen-like U^{91+} . The 0.09 eV uncertainty afflicting the Dirac energy calculation of the $2s$ level in U^{89+} grows to an uncertainty of about 0.55 eV in the calculation of the $1s$ level energy of U^{91+} . This means that a measurement of the energy of the K-shell transitions of hydrogen-like uranium cannot provide a test of the two-loop QED terms to better than this value, as the theoretical value of the Dirac energy needs to be subtracted from the measurement to isolate the two-loop contribution. The two-loop QED energy of the $1s$ level in U^{91+} has been calculated to be $-1.26(33)$ eV by Yerokhin *et al* [46], as given earlier in table 1. The 0.55 eV limitation from the finite size of the uranium nucleus thus limits such tests to about 40%, which is the same limitation we already encountered in using the $2s_{1/2}-2p_{1/2}$ transition energy in U^{89+} for testing two-loop QED.

In fact, using the result from the $2s$ two-loop QED determination we can already determine a value of -1.27 eV for the two-loop QED in $1s$ [44, 47] as we have mentioned in section 3. This value compares very favourably to the value calculated by Yerokhin *et al* [46]. The uncertainty in the derived value depends solely on the assumed uncertainty of the nuclear size of uranium (0.15–0.55 eV)

and the uncertainty of the higher order photon exchange terms (0.33 eV). The experimental uncertainty of the 2005 measurement of the $2s_{1/2}-2p_{1/2}$ transition in U^{89+} is sufficiently low that once the higher order exchange terms are calculated with high accuracy and once the size of the uranium nucleus is more accurately known the 1s two-loop QED term would be tested at the 0.10 eV or 7.5% level.

5. Testing two-loop diagrams in lithium-like bismuth

Tests of two-loop QED in both atomic hydrogen and hydrogen-like uranium are limited by the accuracy with which the finite nuclear size is known. The question arises whether other high-Z ions are better suited for advancing tests of two-loop QED in bound systems.

Highly accurate energy measurements of the $2s_{1/2}-2p_{3/2}$ transition in lithium-like Bi^{80+} have been performed [52]. This measurement was possible because of a near coincidence of the bismuth transition with a calibration line from helium-like chlorine. An experimental accuracy of 0.039 eV was achieved.

At the time of the measurement a Feynman-diagram-based calculation of the $2s_{1/2}-2p_{1/2}$ transition energy was not yet available. However, as such calculations became available, Sapirstein and Cheng pointed out that the Bi^{80+} measurement is the first high-Z measurement to be sensitive to two-loop QED [53]. By subtracting out all first-order contributions they isolated a two-loop QED energy of 0.175 eV. The experimental error bar of 0.039 eV would thus allow a test of the order of 22%.

Bismuth has the advantage that its nuclear size is better known than that of uranium ($\langle r^2 \rangle = 5.521(3)$ fm), and nuclear size effects rapidly decrease with atomic number. In fact, the nuclear size uncertainty limits the calculation of the Dirac energy to 0.03 eV [49], which is less than the experimental error bar. Thus, tests of the two-loop QED terms in bismuth can in principle surpass those performed with uranium. Also, the three-photon exchange term is known to higher accuracy (−0.02 eV) than in the case of uranium (0.07 eV) [49]. A comparison of the Dirac and first-order QED terms performed with different calculational methods [15, 54] thus shows that theory and the nuclear size effects at present limit the test of two-loop QED to about 0.04 eV. This is similar to the experimental error bar. Thus, the accuracy with which two-loop QED can be tested at present in Bi^{80+} is about 35%, as summarized in table 4.

6. Outlook for testing 2-loop diagrams in high-Z ions

The discussion above has demonstrated that tests of two-loop QED in high-Z lithium-like ions are already competitive with measurements utilizing atomic hydrogen. Similar to the experiments involving atomic hydrogen, the experimental precision is no longer a limiting factor. The tests depend strongly on the progress of being able to calculate all the needed terms at the level of the experimental precision achievable to date and on the uncertainty with which the nuclear radii are known.

Table 4. Theoretical contributions [54] to the two-loop QED contribution to the $2s_{1/2}-2p_{3/2}$ transition energy in lithium-like bismuth and comparison with the value inferred from the measurement of Bi^{80+} [52] (in eV).

Contributing term	Value
Theory	
SESE ^a	0.15
VPVP ^b	0.09(3)
S(VP)E ^c	−0.01(2)
SEVP ^d	−0.10
Sum	0.13(4)
Experiment	
Bi^{80+} measurement	0.175(60)

^a Top three diagrams in figure 1(b).

^b Last diagram in the middle of figure 1(b).

^c Bottom three diagrams in figure 1(b).

^d First three diagrams in the middle of figure 1(b).

Assuming that theory continues its pace of remarkable advances, then the progress in testing 2-loop diagrams will depend on either better measurements of nuclear radii or on choosing those ions that already have well known nuclear radii. The most accurate measurement of two-loop QED may thus focus on studying the $2s_{1/2}-2p_{1/2}$ or $2s_{1/2}-2p_{3/2}$ transition in lithium-like Pb^{79+} . Such measurements are likely not to be limited by uncertainties in the finite size of the nucleus. First, this is because the nuclear radii of the stable lead isotopes are eight times better known than those of ^{238}U [51, 55], and second, because the nuclear size effects are only a quarter of those affecting QED in uranium [56]. In other words, the uncertainty in the nuclear size contribution in lead is more than 30 times smaller than that affecting the uranium measurement.

The energy of the $2s_{1/2}-2p_{1/2}$ transition is approximately 230.8 eV [57] and, therefore, falls into the extreme ultraviolet spectral band, similar to the spectral position of the $2s_{1/2}-2p_{1/2}$ transition in uranium. Thus, the measurement could be carried out with high-resolution emission spectroscopy based on a grating spectrometer, e.g. by utilizing the same or a similar apparatus as had been used in the most recent U^{89+} measurement at the SuperEBIT electron beam ion trap facility at Livermore [44, 58, 59]. A picture of that apparatus is shown in figure 4. A measurement of this transition has been made at the experimental storage ring (ESR) at GSI, Darmstadt [43]. However, the experimental precision is not as good as that of the uranium and bismuth measurements already discussed (see figure 2) and thus does not take advantage of the reduced theoretical uncertainties.

If future lead measurement were to achieve an accuracy of 0.005 eV, which is (only) three times better than that of the recent uranium measurement, two-loop QED would be tested at a level well in excess of that achievable in atomic hydrogen.

The $2s_{1/2}-2p_{3/2}$ transition in lithium-like Pb^{79+} can provide a similar window for testing two-loop QED. The 2.6 keV energy of this transition falls into the x-ray band. It has recently been measured utilizing the Tokyo EBIT [60]. The achieved accuracy, however, was 0.10 eV, which, like $2s_{1/2}-2p_{1/2}$ measurement at the ESR, was insufficient for testing two-loop QED.

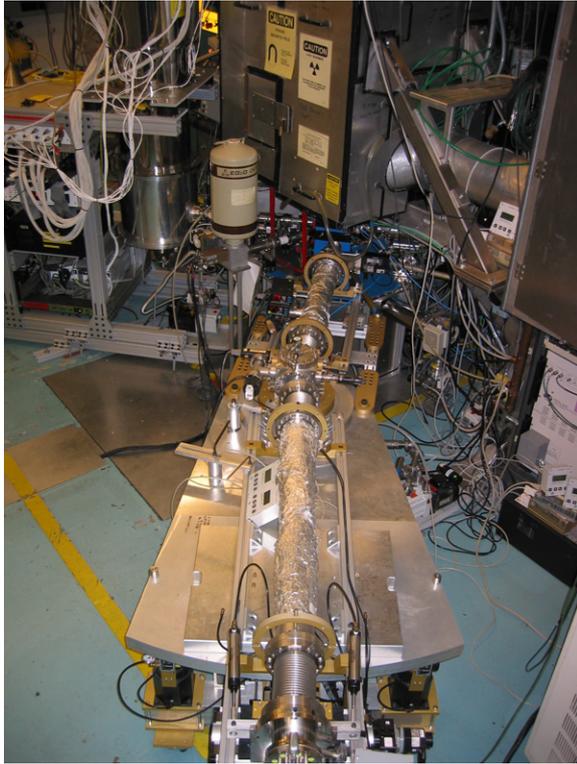


Figure 4. The SuperEBIT electron beam ion trap facility. The high-resolution grating spectrometer used for measuring the $2s_{1/2}-2p_{1/2}$ transition in U^{89+} is shown in the foreground.

7. Hyperfine structure in high-Z ions

The wavefunction of the $1s$ or $2s$ electron overlaps with the nucleus more than that of any other electron. The overlap means that the s electron represents a sensitive probe of nuclear properties. In the two-loop QED measurements discussed above, this probing of nuclear properties had the consequence that the transition energy was sensitive to the size and shape of the distribution of nuclear charge. The s wavefunction is also very sensitive to the size and distribution of the nuclear magnetization. This sensitivity expresses itself as a noticeable perturbation of the standard hyperfine splitting. A complete description of the electron–nuclear interaction and of the nuclear distributions is not only a prerequisite for QED measurements, as our discussion has already shown, but also for the interpretation of atomic parity non-conservation experiments induced by the weak interaction, as detailed in the review by Mårtensson–Pendrill [61].

In atomic hydrogen, the interaction between the magnetic moment of the nucleus and the electron spin of the $1s$ electron splits the $1s$ level into two hyperfine components, as illustrated in figure 5. The magnetic dipole transition between the two components results in the well known 21 cm line in atomic hydrogen. The same interaction splits the $1s$ ground level of hydrogen-like ions by as much as several eV. This means that the photons emitted by some heavy hydrogen-like ions are in the optical wavelength band.

The first measurement of the hyperfine splitting of the $1s$ ground state of a hydrogen-like ion was performed on the ESR

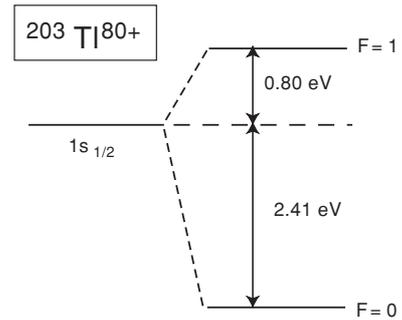


Figure 5. Level diagram of the ground state of hydrogen-like thallium showing the splitting induced by the hyperfine interaction. The nuclear magnetic field ($I = 1/2$) causes a 3.21 eV splitting of the $1s$ ground state of $^{203}\text{Tl}^{80+}$.

heavy-ion storage ring at the GSI Laboratory in Darmstadt. Here, Klaft *et al* used laser excitation to measure the splitting in hydrogen-like $^{209}\text{Bi}^{82+}$ [62]. This measurement was followed by an experiment that employed emission spectroscopy at the SuperEBIT electron beam ion trap at Livermore to determine the hyperfine splitting of the $1s$ level in hydrogen-like $^{165}\text{Ho}^{66+}$ [63]. Laser excitation was used in a third measurement at GSI, Darmstadt, to determine the splitting in $^{207}\text{Pb}^{81+}$ [64]. The passive emission spectroscopy technique was subsequently used at Livermore to measure the $1s$ splitting in four more hydrogen-like ions, namely, $^{185}\text{Re}^{74+}$, $^{187}\text{Re}^{74+}$, $^{203}\text{Tl}^{80+}$ and $^{205}\text{Tl}^{80+}$ [65, 66]. A summary of the measured values is given in table 5. One should note that there exists an earlier method of measuring the hyperfine interaction, which relies on creating muonic atoms. For example, measurements of the $1s$ hyperfine splitting in muonic thallium determined 2.34(8) for ^{203}Tl and 2.30(2) keV for ^{205}Tl [67]. Measurements of the electronic hyperfine splitting carried out at GSI and LLNL are, however, about two orders of magnitude more accurate than the muonic measurements, and the muonic measurements are not competitive (this, by the way, also holds for muonic QED measurements).

Table 5 also lists theoretical values. There is a surprisingly large discrepancy between the measured values and theory. For example, the measurement of the hyperfine splitting of the $1s$ level in $^{209}\text{Bi}^{82+}$ produced a value of 5.0840(8) eV, while calculations achieved, for example, values of 5.050(8) eV, 5.058(8) eV, 5.054(7) eV, 5.100(27) and 5.0933(8) eV [19, 68, 69]. When expressed in wavelengths, i.e. the actual quantity measured in the experiments, the differences are even more striking. For example, the calculated transition wavelengths for the hyperfine transition in hydrogen-like ^{205}Tl are 3786 Å [70], 3802 Å [71] and 3849 Å [72]. These vary by about 60 Å amongst each other and differ between -36 and $+27$ Å from the measured value of 3821.84 ± 0.34 Å [66]. Clearly, it is difficult to even identify these transitions in a spectrum or find the proper overlap with a tuned laser beam based on these predictions.

Figure 6 shows percentage differences between theory and measurement. A difference of 0.5–1.0% corresponds to 20–50 Å. The differences are largest for those isotopes far from the doubly magic nucleus ^{208}Pb . This suggests that inaccurate knowledge of the nuclear properties, especially knowledge

Table 5. Comparison between measured and calculated 1s hyperfine transition energies (in eV).

Year	Facility	Reference	Ion	Experiment	Theory [75]	Theory [76]	Theory [72]
1994	ESR	[62]	$^{209}\text{Bi}^{82+}$	5.0840 ± 0.0008	5.0920 ± 0.0150	5.100 ± 0.027	5.0552
1996	SuperEBIT	[63]	$^{165}\text{Ho}^{66+}$	2.1645 ± 0.0006	2.1863 ± 0.0072	2.188 ± 0.007	2.1546
1998	ESR	[64]	$^{207}\text{Pb}^{81+}$	1.2159 ± 0.0002	1.2120 ± 0.0110	1.215 ± 0.005	1.2099
1998	SuperEBIT	[65]	$^{185}\text{Re}^{74+}$	2.7190 ± 0.0018	2.7480 ± 0.0100	2.749 ± 0.010	2.7050
1998	SuperEBIT	[65]	$^{187}\text{Re}^{74+}$	2.7450 ± 0.0018	2.7760 ± 0.0100	2.776 ± 0.010	2.7306
2001	SuperEBIT	[66]	$^{203}\text{Tl}^{80+}$	$3.213\ 51 \pm 0.000\ 25$	3.2290 ± 0.0170	3.228 ± 0.018	3.1954
2001	SuperEBIT	[66]	$^{205}\text{Tl}^{80+}$	$3.244\ 09 \pm 0.000\ 29$	3.2610 ± 0.0180	3.259 ± 0.018	3.2213

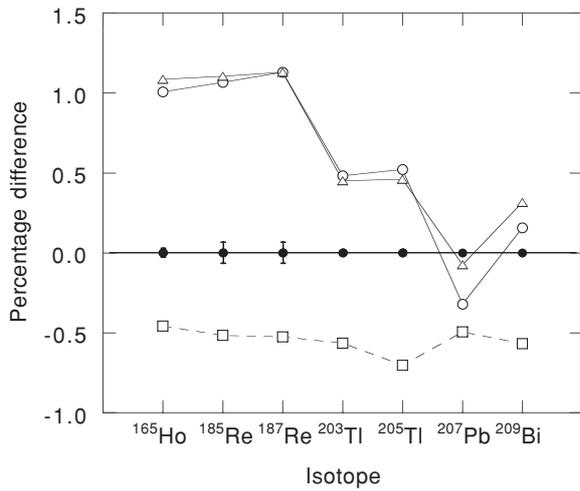


Figure 6. Fractional difference of the calculated and measured 1s hyperfine splitting. All values are referenced to the experimental data (solid circles with errors). Theory values are from [75] (open circles), [72] (open squares) and [76] (open triangles).

of the nuclear magnetization radius, could be the cause for the discrepancies [73]. In fact, much of the variation among the different theoretical results stems from the uncertainties in the calculated values of the nuclear magnetization distribution. The calculation of the nuclear magnetization distribution, which expresses itself via the so-called Bohr–Weisskopf effect, typically relies on solving the Schrödinger equation for a single unpaired nucleon. This single-particle model (SPM) is most accurate for nuclei next to a nucleus with a closed shell of nucleons, such as the two nuclei next to ^{208}Pb . The model becomes increasingly suspect away from the doubly magic ^{208}Pb nucleus. Tomaselli *et al* developed the dynamic correlation model (DCM) [74] to improve on the single-particle model. Their results are closer to the measurements in some cases, but overall differ from the measurements by a nearly constant -0.5% , i.e. they differ even for Pb and Bi [72].

The difference between the measured value and theory may also be caused by errors in the uncertainties stated in the tabulated values of the nuclear magnetic moments. In fact, Gustavsson and Mårtensson-Pendrill [77] have shown that the nuclear magnetic moments are generally less accurate than stated. However, if one assumes that the entire difference between the single-particle model and experimental value is due to inaccuracies in the nuclear magnetization, the experimental value can be used to solve for the magnetization distribution. This was done in [66, 65, 78]. The values

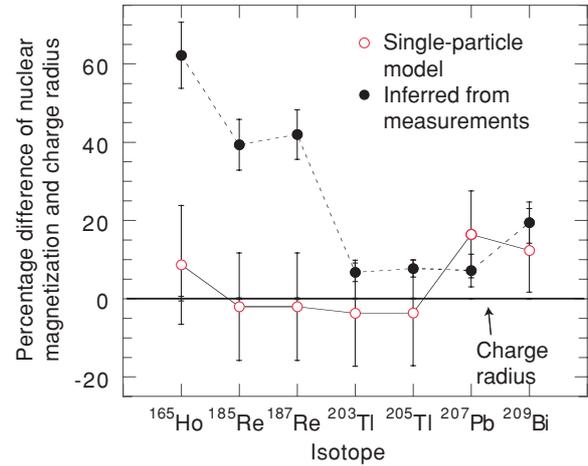


Figure 7. Percentage difference of the calculated and measured nuclear magnetization radius and the known values of the nuclear charge radius (with error bars).

Table 6. Calculated and inferred nuclear magnetization radii $\delta(r_m^2)^{1/2}$ (in fm) for ^{203}Tl and ^{205}Tl [66].

Ion	SPM	Experiment
^{203}Tl	5.27	5.83 ± 0.14
^{205}Tl	5.27	5.89 ± 0.14

obtained for the nuclear magnetization radii $\delta(r_m^2)^{1/2}$ for the two ^{203}Tl and ^{205}Tl nuclei were more than 10% larger than predicted by theory, as seen from table 6. This increased nuclear magnetization radius is expected given that the thallium nuclei are already several nucleons away from a doubly closed shell. The earlier measurements of the two $^{185}\text{Re}^{74+}$ and $^{187}\text{Re}^{74+}$ nuclei, which are even further away from the doubly closed shell ^{208}Pb nucleus resulted in nuclear magnetization radii that were 44% larger than calculated. This trend can be seen in figure 7, which gives an overview of the difference between the magnetization radius calculated via the single-particle model and the values inferred from the experimental data. The figure shows that the calculated magnetization radius is close to that of the measured charge radius for nuclei away from the magic ^{208}Pb nucleus; surprisingly, the biggest deviation occurs for the two nuclei next to ^{208}Pb .

In figure 8 we show the difference of the Bohr–Weisskopf contribution inferred from the experimental data and from different calculations. The Bohr–Weisskopf correction calculated with the single-particle model [75, 79] does well near the doubly magic nucleus, but it systematically

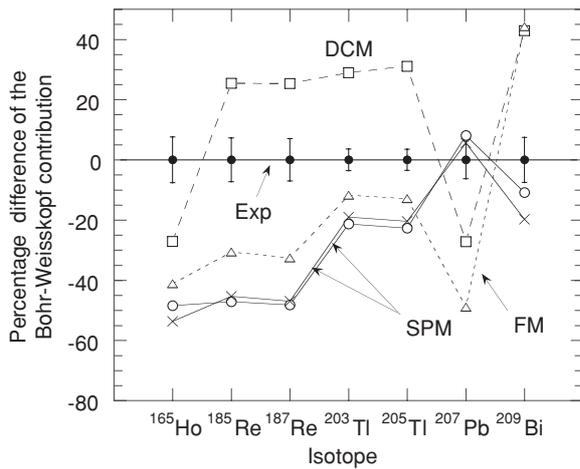


Figure 8. Fractional difference of the calculated and measured Bohr–Weisskopf contributions. All values are referenced to the experimental data (solid circles with error bars). Theory values are from [75] (open circles), [79] (crosses), [72] open squares and open triangles [80].

underestimates the values inferred from the experimental data away from the doubly magic nucleus. The differences between the inferred Bohr–Weisskopf corrections and those calculated with the dynamic correlation [72] and a Fermi distribution-like model [80] are also shown.

Calculations of the hyperfine structure have been extended to the 2s electron of lithium-like ions [73, 81, 82]. It was shown by Shabaev *et al* [73] that the Bohr–Weisskopf effect and thus the uncertainty associated with this effect cancels out if both the 1s splitting ($\Delta E^{(1s)}$) of the hydrogen-like ion and the 2s splitting ($\Delta E^{(2s)}$) of the lithium-like ion are known and the appropriate difference is formed. That difference is given by

$$\Delta' E = \Delta E^{(2s)} - \zeta \Delta E^{(1s)}. \quad (1)$$

For bismuth Shabaev *et al* calculated $\zeta = 0.16885$ and $\Delta' E = -0.06127$ eV.

So far only one measurement for the 2s splitting has been reported. This measurement determined the hyperfine splitting of the 2s level in $^{209}\text{Bi}^{80+}$ and was made on SuperEBIT at Livermore [83]. The value obtained was $\Delta E^{(2s)} = 0.820(26)$ eV. This measurement results in a value of $\Delta' E = -0.038(26)$ eV, which is too imprecise to test theory. Additional measurements are underway both at the heavy-ion accelerator at GSI, Darmstadt [84], and SuperEBIT at Livermore. A measurement studying the hyperfine splitting of the 2s level at the SuperEBIT was recently reported [85]; however, the analysis is still in progress.

8. Conclusion

Measurements of QED in high-Z ions have clearly come of age and are competitive with those of neutral atoms using laser spectroscopy. In particular, this is true for measurements of high-Z lithium-like ions, the results of which are unrivaled by those of high-Z hydrogen-like systems. The experimental accuracy now exceeds the limitations imposed by the uncertainties in the finite size of the nuclei under study. This is true in particular for hydrogen and uranium, where no

further progress can be expected until the nuclear charge radii are known with higher accuracy.

The unprecedented advances in testing two-loop QED in high-Z systems were made possible by (1) the availability of highly charged lithium-like ions in the SuperEBIT electron trap at Livermore and high-resolution spectroscopic instrumentation matched to this source and (2) the progress made in theory, which can evaluate Feynman diagrams for lithium-like ions with high accuracy. Our discussion has shown that other high-Z ions may lend themselves to tests of two-loop QED that may greatly exceed those afforded by measurements of hydrogen and uranium. Thus, the field has not yet reached its ultimate limits. These new tests will require measurements with yet higher experimental accuracy as well as theoretical calculations that improve upon the current state-of-the-art of determining the three-photon exchange term and other terms entering at this level.

The theoretical determinations of the hyperfine splitting in hydrogen-like ions are still not yet adequate to account for the observations. This state of affairs is especially unsettling given that hydrogen-like ions are the ‘Rosetta Stone’ of quantum mechanics. Much of the blame may be laid upon the fact that the finite extent of the magnetization radii is only poorly known as well as upon the fact that the nuclear magnetic moments are probably more poorly known than the experimental error bars suggest. However, one should note that the magnetic fields in these nuclei are the highest such fields found in nature, and there is still the possibility that there is as of yet unknown new physics associated with such high magnetic fields. A true test of the calculations will come when the hyperfine splitting of both the 1s and the 2s levels will have been measured in the same isotope so that the effects of the finite magnetization radius can be eliminated. This field has also not yet reached its ultimate potential and surprises may still be possible.

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References

- [1] van Dyck R S Jr, Schwinger P B and Dehmelt H G 1987 *Phys. Rev. Lett.* **59** 26
- [2] Odom B, Hanneke D, Urso B D and Gabrielse G 2006 *Phys. Rev. Lett.* **97** 030801
- [3] Hughes V W and Kinoshita T 1999 *Rev. Mod. Phys.* **71** S133
- [4] Gabrielse G, Hanneke D, Kinoshita T, Nio M and Odom B 2006 *Phys. Rev. Lett.* **97** 030802
- [5] Niering M *et al* 2000 *Phys. Rev. Lett.* **84** 5496
- [6] Udem T, Huber A, Gross B, Reichert J, Prevedelli M, Weitz M and Hänsch T W 1997 *Phys. Rev. Lett.* **79** 2646
- [7] Fischer M *et al* 2004 *Phys. Rev. Lett.* **92** 230802
- [8] Blatt S *et al* 2008 *Phys. Rev. Lett.* **100** 104801
- [9] Flambaum V V and Dzuba V A 2009 *Can. J. Phys.* **87** 25
- [10] Weitz M, Huber A, Schmidt-Kaler F, Leibfried D and Hänsch T W 1994 *Phys. Rev. Lett.* **72** 328
- [11] Schwob C, Jozefowski L, de Beauvoir B, Hilico L, Nez F, Julien L, Biraben F, Aefel O and Clairon A 1999 *Phys. Rev. Lett.* **82** 4960

- [12] de Beauvoir B, Schwob C, Acef O, Jozefowski L, Hilico L, Nez F, Julien L, Clairon A and Biraben F 2000 *Eur. Phys. J. D* **12** 61
- [13] Gumberidze A *et al* 2005 *Phys. Rev. Lett.* **94** 223001
- [14] Beiersdorfer P 2009 *Can. J. Phys.* **87** 9
- [15] Cheng K, Chen M H, Johnson W R and Sapirstein J 2008 *Can. J. Phys.* **86** 33
- [16] Blundell S A 2009 *Can. J. Phys.* **87** 55
- [17] Chen M H, Cheng K T, Beiersdorfer P and Sapirstein J 2003 *Phys. Rev. A* **68** 022507
- [18] Beiersdorfer P, Träbert E, Chen H, Chen M-H, May M J and Osterheld A L 2003 *Phys. Rev. A* **67** 052103
- [19] Sapirstein J and Cheng K 2008 *Can. J. Phys.* **86** 25
- [20] Drake G W F 2002 *Can. J. Phys.* **80** 1195
- [21] Pachucki K and Jentschura U D 2003 *Phys. Rev. Lett.* **91** 113005
- [22] Zelevinsky T, Farkas D and Gabrielse G 2005 *Phys. Rev. Lett.* **95** 203001
- [23] Story M C G C H and Hessels E A 2000 *Phys. Rev. Lett.* **84** 3274
- [24] Castilleja J, Livingston D, Sanders A and Shiner D 2000 *Phys. Rev. Lett.* **84** 4321
- [25] Giusfredi G, Cancio Pastor P, De Natale P, Mazzotti D, de Mauro C, Fallani L, Hagel G, Krachmalnicoff V and Inguscio M 2005 *Can. J. Phys.* **83** 301
- [26] Karshenboim S G 1999 *Can. J. Phys.* **77** 241
- [27] Pachucki K 1994 *Phys. Rev. Lett.* **72** 3154
- [28] Weitz M, Huber A, Schmidt-Kaler F, Liebfried D, Vassen W, Zimmermann C, Pachucki K, Hänsch T W, Julien L and Biraben F 1995 *Phys. Rev. A* **52** 2664
- [29] Simon G G, Schmitt C, Borkowski F and Walther V H 1980 *Nucl. Phys. A* **333** 381
- [30] Pachucki K, Liebfried D, Weitz M, Huber A, König W and Hänsch T W 1996 *J. Phys. B: At. Mol. Opt. Phys.* **29** 177
- [31] Hand L N, Miller D J and Wilson R 1963 *Rev. Mod. Phys.* **35** 335
- [32] Höhler G, Höhlerb G, Pietarinenb E, Sabba-Stefanescu I, Borkowski F, Simon G G, Walther V H and Wendling R D 1976 *Nucl. Phys. B* **114** 505
- [33] Mergell P, Meißner U G and Drechsel D 1996 *Nucl. Phys. A* **596** 367
- [34] Rosenfelder R 2000 *Phys. Lett. B* **479** 381
- [35] Sick I 2007 *Can. J. Phys.* **85** 409
- [36] Lupton J H, Dietrich C J H D D, Stewart R E and Ziocok K P 1994 *Phys. Rev. A* **50** 2150
- [37] Stöhlker T *et al* 1993 *Phys. Rev. Lett.* **71** 2184
- [38] Beyer H F *et al* 1995 *Z. Phys. D* **35** 169
- [39] Stöhlker T *et al* 2000 *Phys. Rev. Lett.* **85** 3109
- [40] Schweppe J *et al* 1991 *Phys. Rev. Lett.* **66** 1434
- [41] Beiersdorfer P, Knapp D, Marrs R E, Elliott S R and Chen M H 1993 *Phys. Rev. Lett.* **71** 3939
- [42] Beiersdorfer P, Osterheld A, Elliott S R, Chen M H, Knapp D and Reed K 1995 *Phys. Rev. A* **52** 2693
- [43] Brandau C *et al* 2003 *Phys. Rev. Lett.* **91** 073202
- [44] Beiersdorfer P, Chen H, Thorn D B and Träbert E 2005 *Phys. Rev. Lett.* **95** 233003
- [45] Yerokhin V A and Shabaev V M 2001 *Phys. Rev. A* **64** 062507
- [46] Yerokhin V A, Indelicato P and Shabaev V M 2003 *Phys. Rev. Lett.* **91** 073001
- [47] Beiersdorfer P 2006 *Radiat. Phys. Chem.* **75** 1757
- [48] Andreev O Y, Labzowsky L N, Plunien G and Soff G 2001 *Phys. Rev. A* **64** 042513
- [49] Yerokhin V A, Indelicato P and Shabaev V M 2006 *Phys. Rev. Lett.* **97** 253004
- [50] Zumbro J D, Shera E B, Tanaka Y, Bemis C E Jr, Nauman R A, Hoehn M V, Reuter W and Steffen R M 1984 *Phys. Rev. Lett.* **53** 1888
- [51] Angeli I 2004 *At. Data Nucl. Data Tables* **87** 185
- [52] Beiersdorfer P 1998 *Proc. of the US-Japan Workshop and Int. Seminar on Plasma Polarization Spectroscopy (Kyoto, Japan, January 26–28, 1998)* NIFS-PROC-37 ed T Fujimoto and P Beiersdorfer (National Institute for Fusion Science, Toki, Gifu, 1998) pp 67–89
- [53] Sapirstein J and Cheng K T 2001 *Phys. Rev. A* **64** 022502
- [54] Yerokhin V A, Indelicato P and Shabaev V M 2007 *Can. J. Phys.* **85** 521
- [55] Fricke G, Bernhardt C, Heilig K, Schaller L A, Schellenberg L, Shera E B and de Jager C W 1995 *At. Data Nucl. Data Tables* **60** 177
- [56] Johnson W R and Soff G 1985 *At. Data Nucl. Data Tables* **33** 405
- [57] Kim Y-K, Baik D H, Indelicato P and Desclaux J P 1991 *Phys. Rev. A* **44** 148
- [58] Beiersdorfer P, Magee E W, Träbert E, Chen H, Lepson J K, Gu M-F and Schmidt M 2004 *Rev. Sci. Instrum.* **75** 3723
- [59] Beiersdorfer P 2008 *Can. J. Phys.* **86** 1
- [60] Zhang X, Nakamura N, Chen C, Andersson M, Liu Y and Ohtani S 2008 *Phys. Rev. A* **78** 032504
- [61] Mårtensson-Pendrill A-M 2008 *Can. J. Phys.* **86** 99
- [62] Klafit I *et al* 1994 *Phys. Rev. Lett.* **73** 2425
- [63] López-Urrutia J R C, Beiersdorfer P, Savin D W and Widmann K 1996 *Phys. Rev. Lett.* **77** 826
- [64] Seelig P *et al* 1998 *Phys. Rev. Lett.* **81** 4824
- [65] Crespo López-Urrutia J R, Beiersdorfer P, Widmann K, Birkett B B, Mårtensson-Pendrill A-M and Gustavsson M G H 1998 *Phys. Rev. A* **57** 879
- [66] Beiersdorfer P *et al* 2001 *Phys. Rev. A* **64** 032506
- [67] Backe H *et al* 1972 *Nucl. Phys. A* **189** 472
- [68] Persson H, Schneider S M, Greiner W, Soff G and Lindgren I 1996 *Phys. Rev. Lett.* **76** 1433
- [69] Sunnergren P, Persson H P, Salomonson S, Schneider S M, Lindgren I and Soff G 1998 *Phys. Rev. A* **58** 1055
- [70] Shabaev V M 1994 *J. Phys. B: At. Mol. Opt. Phys.* **27** 5825
- [71] Gustavsson M G H, Forssén C and Mårtensson-Pendrill A-M 2000 *Hyperfine Interact.* **127** 347
- [72] Tomaselli M, Kühl T, Nörtershäuser W, Borneis S, Dax A, Marx D, Wang H and Fritzsche S 2002 *Phys. Rev. A* **65** 022502
- [73] Shabaev V M, Artemyev A N, Yerokhin V A, Zherebtsov O M and Soff G 2001 *Phys. Rev. Lett.* **86** 3959
- [74] Tomaselli M, Schneider S M, Kankeleit E and Kühl T 1995 *Phys. Rev. C* **51** 2989
- [75] Gustavsson M G H 2000 *PhD thesis* Physics and Engineering Physics, Göteborg University and Chalmers University of Technology
- [76] Beier T 2000 *Phys. Rep.* **339** 79
- [77] Gustavsson M G H and Mårtensson-Pendrill A-M 1998 *Phys. Rev. A* **58** 3611
- [78] Beiersdorfer P, Crespo López-Urrutia J R, Utter S B, Träbert E, Gustavsson M, Forssén C and Mårtensson-Pendrill A-M 2003 *Nucl. Instrum. Methods B* **205** 62
- [79] Shabaev V M, Tomaselli M, Kühl T, Artemyev A N and Yerokhin V A 1997 *Phys. Rev. A* **56** 252
- [80] Boucard S and Indelicato P 2000 *Eur. Phys. J. D* **8** 59
- [81] Shabaev V M, Shabaeva M B and Tupitsyn I I 1995 *Phys. Rev. A* **52** 3686
- [82] Sapirstein J and Cheng K T 2001 *Phys. Rev. A* **63** 032506
- [83] Beiersdorfer P, Osterheld A, Scofield J, Crespo López-Urrutia J and Widmann K 1998 *Phys. Rev. Lett.* **80** 3022
- [84] Winters D F A, Vogel M, Segal D M, Thompson R C and Nörtershäuser N C 2007 *Can. J. Phys.* **85** 403
- [85] Beiersdorfer P, Träbert E, Brown G V, Chen H, Clementson J and Thorn D 2007 Abstract Book of 25th *International Conference on Photonic, Electronic and Atomic Collisions* (Freiburg, Germany, 25–31 July 2007)